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## Abstract

Enrolment rates to higher education reveal quite large variation over time which cannot be explained by productivity shocks alone. We develop a human capital investment model in an overlapping generations framework that features endogenous fluctuations in the demand for education. Agents are heterogeneous in their beliefs about future wage differentials. An evolutionary competition between the heterogeneous beliefs determines the fraction of the newborn generation having a certain belief. Costly access to information on the returns to education induces agents to use potentially destabilizing backward looking prediction rules. Only if previous generations experience regret about their human capital investment decisions, agents will choose a more sophisticated prediction rule that dampens the cycle. Access to information becomes key for stable flows to higher education.

Keywords: expectations, human capital investment, endogenous fluctuations, inter-generational spill-overs, evolutionary dynamics, bifurcation analysis
JEL-classification: C60, E32, J24

## Zusammenfassung

Einschreibungen an Fachhochschulen und Universitäten weisen starke zyklische Schwankungen auf, die nicht allein durch Produktivitätsschocks erklärt werden können. Es wird ein Humankapitalmodell mit überlappenden Generationen vorgestellt, das die Eigenschaft besitzt, Zyklen in der Nachfrage nach Bildung endogen zu erklären. Im Modell sind die Akteure heterogen in Bezug auf ihre Voraussagen über zukünftige Lohndifferentiale. Ein evolutionärer Wettbewerb unter den Voraussagemethoden bestimmt den Anteil der Akteure, der ein bestimmtes Prognoseinstrument verwendet. Da der Zugang zu Informationen über zukünftige Humankapitalrenditen mit Kosten verbunden ist, weichen die Akteure auf vergangenheitsorientierte Voraussagemethoden aus, die destabilisierend wirken können. Nur dann, wenn frühere Generationen die Art und Weise wie sie prognostizierten bereuen, werden die Akteure kompliziertere, zukunftsgerichtete, Prognoseinstrumente wählen, welche die Zyklen dämpfen. Damit werden leicht verfügbare Informationen über Arbeitsmarktentwicklungen zur Schlüsselvariablen für konstante Zugänge zu Fachhochschulen und Universitäten, aber auch anderen Bildungsträgern.

# Contents

# 1 Introduction

Enrollment rates to higher education vary over time. This holds true for many countries and educational fields. College enrollment rates in the U.S. fluctuated between 37% and 47% in the years from 1968 to 1988, where controlling for individual and regional effects does not cancel out the dynamics (Dellas and Sakellaris 1995). Time series for Sweden indicate a variation of college enrollment rates of 20 to 24 year old men between 34% and 46% in the years from 1963 to 1991 (Topel 1997). For engineering degrees at universities, Germany was confronted with enrollment rates between 8% and 12% of all graduates from upper secondary schooling from 1975 to 1998. For those students enrolling to non-university type of higher education, rates changed between 32% and 52% of all graduates with a specialized upper secondary degree (Neugart 2001). Freeman (1976b) shows for various fields of study the dynamics of enrollments to U.S. colleges.[1]

There is a large literature aiming to explain changes in the demand for higher education with variations in expected relative wages. Approaches taken so far may be divided according to the way how agents' expectations are modelled.

Backward looking expectations postulate that students, entering higher education programs, form expectations on future relative wages using past experiences. What counts in the 'cost-benefit' analysis that underlies the human capital investment decision is actual wages, or most recently paid wages, relative to what could be earned without investment into schooling. There is time series evidence, usually on the basis of dynamic regression models, that finds enrollment rates driven by backward looking expectations (Freeman 1975a and 1975b, Freeman 1976a, Borghans et al. 1996, Duchesne and Nonneman 1998, Quinn and Price 1998, Card and Lemieux 2000).

On the other hand, one may employ rational expectations. This assumes that agents make unbiased forecasts on future relative wages. Zarkin (1983, 1985) and Siow (1984) have estimated rational expectations models for enrollments of teachers and lawyers, respectively. Both find support for the hypothesis of rationally forecasting agents.

Reviewing the literature on the demand for education Freeman (1986) concludes, that, when assuming backward looking expectations, the internal market structure can generate cobweb type of 'ups' and 'downs' in enrollment rates. However, the estimated models would imply damped oscillations. To arrive at continued oscillations one would have to recur to large shocks. Such shocks are also needed if one wants to explain cyclical behavior in enrollment rates under the assumption of rational expectations. Clearly, both approaches shift the attention to factors that lie outside the labor market.

The aim of our paper is to offer an endogenous explanation of enrollment dynamics. Even though we focus on internal forces, we do not think that exogenous shocks shifting the demand side of the labor market should be ruled

[1] A collection of data on enrollments for a large range of countries can be found on the homepage of the UNESCO *http://unescostat.unesco.org/en/stats/stats0.htm*. The compendium by Titze et al. (1986, 1993) contains time series on enrollments for all major subjects for Germany going back to 1820. The fluctuations shown there are quite striking.

out as a potential cause. However, the role of exogenous shocks may be less important than needed to explain continuing large fluctuations in enrollments.

We develop a human capital investment model with overlapping generations. Agents, deciding on whether to invest into schooling, use either a costless backward looking predictor or incur costs for making a more sophisticated prediction on future relative wages. The predicted wage rate then determines the schooling decision. An evolutionary competition between the different forecasting rules determines the fraction of the agents in the newborn generation that uses a certain forecasting rule. This competition is driven by the performance of backward looking agents of the previous generation. Thus, inter-generational spill-overs guide the new born generation in their choice of the predictor rule. They are more likely to predict with a sophisticated mechanism, given that access to information on returns to human capital investments is costly, if the previous generation did poorly with the backward looking predictor. The dynamics of the model are such that backward looking expectations may destabilize enrollments, very much in the the spirit of an unstable cobweb. However, oscillations will be bounded as agents switch to a forward looking predictor if the previous, mostly naive population, regrets its schooling decision.

The policy conclusion from the analysis is that easing access to information on human capital investments can stabilize flows to education. This may help avoiding situations where universities are flooded with students so that capacity constraints are hit, unless costly excess capacity is held. Policies that stabilize flows to universities may also raise the quality of education that generally suffers from overcrowded universities.

Evolutionary belief formation is key in our augmented human capital model with which we try to give an endogenous explanation for cycles in the demand for higher education. There has been a number of recent contributions to the literature that consider an evolutionary competition between heteregeneous beliefs. One strand of the literature uses genetic algorithms to model this evolutionary competition. Applications of this approach to models of the cobweb type can be found in Arifovic (1994), Dawid and Kopel (1998) and Franke (1998). The latter finds, as do we in our analysis on dynamics in the demand for education, that increased evolutionary pressure destabilizes the economy. Another strand of the literature uses the methodology from dynamical systems theory to study the evolutionary competition between heterogeneous beliefs in more stylized and less computationally intensive models. Our paper falls in this strand of the literature and is closely related to the work of Brock and Hommes (1997) who show that an evolutionary competition between heterogeneous beliefs in a cobweb model might lead to endogenous fluctuations. Competition between trading strategies or behavioral rules, and the choice of one over the other based on past performance, has also been shown to generate complicated dynamics in models of financial markets (see for example Brock and Hommes 1998, Lux 1998, Chiarella 1992) and Cournot competition (Droste, Hommes and Tuinstra 2002). That the inclusion of heterogenous beliefs in models on market activity can reveal interesting results in terms of the dynamical properties of market systems is also the subject of Barucci (1999), who shows that introducing heterogeneous beliefs may lead to a smaller stability region in forward looking economic models as compared to single agent models. Moreover, just as in our model, in his heterogeneous beliefs model flip and Neimark-Sacker or Hopf bifurcations occur.

The notion of 'not-so-rational' agents in our model relates to survey evidence for European and U.S. college students on motives for studying (Brunello, Lucifora and Winter-Ebmer 2001, and Betts 1996, respectively). The former find, based on survey data of more than 6000 college students, that the average expected college wage gains are larger than estimated actual wage gains. In their study overoptimistic students expect to earn far more than 10% over what they actually will. The latter, with U.S. data, also finds overestimation of wage gains from college attendance of approximately 10%. Both results seem to indicate that the enrollment decision is not based on a rational forecast. However, as it is evidence for a specific year in each data set only, it does not yield insight into whether beliefs change over time. That gap may be filled with simulation studies as they can be found in (Borghans et al. 1996 and Neugart 2001). In both papers a model for enrollments to higher education based on backward looking expectations is estimated. The model is then used to generate a time series on hypothetical enrollments, that is, enrollments had students known the actual wage at the time of graduation. Comparing hypothetical and actual enrollments gives information on how many students would have or would not have chosen schooling had they known the future labor market status. All we want to state here with respect to these findings is that the number of students making the 'wrong' decisions varies over time quite substantially. Certainly, not having the right forecast may be due to frequent shocks to the market. However, those would have to be rather marked.

The inter-generational spill-over effect in the belief formation of our agents can be put into the context of other models where the schooling decision is also a function of the social environment. Externalities, so far investigated, were either intra- or inter-generational. Bala and Sorger (1998, 2001) study the former in a spatio-temporal set-up. There, returns on human capital investments do not only depend on the agent's effort but also on the human capital endowment of so called peer groups. Good neighborhoods, or skilled socially relevant fellow agents, may have a positive impact on human capital accumulation and vice versa. One important policy implication from such models is that with intra-generational spill-overs one may observe an endogenous formation of skilled and un-skilled agents over space. Removing credit constraints does not necessarily shut off stratification. Inter-generational spill-overs play central roles in Orazem and Tesfatsion (1997) and de la Croix (2001). The overlapping generations model of Orazem and Tesfatsion (1997) consists of multiple dynasties. In lack of an adequate predictor on future wages, students learn from their parents whether it pays to invest into schooling. Higher observed returns on education of the parents induces their children to increase effort. Orazem and Tesfatsion (1997) study the impact of taxing adults on the schooling decision of their children. In de la Croix (2001) a new born generation inherits aspirations and human capital of the previous generation. While a high human capital level serves as a positive incentive to invest in ones own education, high aspirations carry the opposite sign. The composite effect on the current generations' consumption behavior may be such that consumption is too high, given current productivity growth. The new generation may not invest into human capital enough to keep the economy growing. Depending on the relative strength of the two externalities the economy may run into a poverty trap or cycle along a growth path.

Studying heterogenous beliefs in the schooling decision may also contribute to the interpretation of estimates on schooling behavior. Manski (1993) already emphasized the lack of evidence that prevailing assumptions on expectations are correct. Moreover, he indicated that incorrect assumptions on expectations may lead to biases in empirical estimates of schooling choices. The reason is that it is impossible to tell how students make their schooling decisions from observing schooling choices as long as one does not know how students perceive the returns to schooling. Observing the choices and the returns only allows to determine a function for schooling choices, if one assumes a specific expectation rule. If the assumption on expectations is false, however, so will be the estimates of the schooling choice. Clearly, a better understanding of how expectations are formed is required.

Our paper is organized in the following way. We first sketch a general model with respect to consumers, firms and evolutionary belief formation when agents live for two periods and generations overlap. The next section develops a more specific model. It is assumed that agents choose between a naive and a rational predictor. A numerical example is introduced for which the dynamic properties are studied analytically and with computational methods. Section 4 presents an evolutionary competition between two other forecasting rules: steady state forecasters – agents that know the long run wage differential between high-skilled and low-skilled work – and 'adaptive' forecasters. Dynamical features, similar to those found in Section 3 emerge in this framework, but also other bifurcation routes are possible. Section 5 concludes. Proofs of the main results can be found in the Appendix.

# 2 The Model

## 2.1 The firms

The economy consists of a sector $H$ and a sector $L$ which produce the same commodity. Sector $H$ employs high-skilled labor, sector $L$ employs low-skilled labor. We assume that both sectors use labor as the only factor of production. Firms in sector $L$ produce according to a constant returns to scale production technology, implying a constant real wage rate for low-skilled labor which we normalize to unity. Demand for low-skilled labor is perfectly elastic at this real wage of 1. Firms in sector $H$ produce according to a concave production technology $f(l)$. Profit maximization yields a decreasing demand function for high-skilled labor $l^d(w_t)$ as the solution to $f'(l) = w_t$, where $w_t$ denotes the real wage rate for high-skilled labor in period $t$.

## 2.2 The consumers

We assume an overlapping generations structure, where in each period $t$ a continuum of agents of mass one is born that lives for two periods. Agent $i$ has private costs of effort $e_i$ for investing into education. These effort costs are distributed according to some distribution function $F$ with support $[0,1]$. Agents have one time unit in each period of their life. They can use part of their time endowment in the first period of their life for investing into education. If they invest into education they acquire certain skills which allow them to work in sector $H$ in the second period of their life. If they do not invest in education they will have to work in sector $L$ all of their life. The choice whether or not to invest into education is made in the beginning of the first period of their life, right after they observe the market clearing wage rates in sectors $H$ and $L$.

An agent born in period $t$ has a lifetime utility function

$$U\left(c_t, c_{t+1}\right),$$

where $c_t$ and $c_{t+1}$ denote consumption in period $t$ and $t+1$, respectively. We will assume that this utility function is monotonic, strictly concave and twice differentiable. Furthermore, we assume that marginal utility of consumption in the first period approaches infinity as consumption in the first period approaches zero, that is $\lim_{c_t \to 0} \frac{\partial U}{\partial c_t} = \infty$.

If agent $i$ decides to invest in education his consumption levels will be $c_t = 1 - e_i$ and $c_{t+1} = w_{t+1}$. Effort costs can therefore be interpreted as the fraction of time that a young agent has to spend on education to acquire the skills for working in the high-skill sector in the second period of his life. If an agent does not invest in education and decides to work in sector $L$ all of his life, he incurs no effort costs. His consumption will then be $c_t = c_{t+1} = 1$. Notice that we assume that agents cannot transfer income from one period to the next.

Agent $i$ will invest in education if he expects lifetime utility to be at least as large as when he would not be investing into education. Hence, investment into human capital will occur if and only if

$$U\left(1 - e_i, w_{t+1}^e\right) \geq U\left(1,1\right),$$

where $w_{t+1}^e$ refers to expectations, formed in period $t$, on real wages to be paid in sector $H$ in $t+1$. By equating these utility levels we find the *marginal effort level*, that is the effort level for which an agent is indifferent between investing and not investing into human capital. Denote this marginal effort level by $e_t^* = e\left(w_{t+1}^e\right)$. Note that, by our assumption on marginal utility, this marginal effort level always exists for $w_{t+1}^e > 1$. Since utility is decreasing in the effort level all agents with $e_i < e\left(w_{t+1}^e\right)$ will invest in schooling. As the effort level is distributed on $[0,1]$, the fraction of the generation born at time $t$, with wage expectation $w_{t+1}^e$, that invests in education will be equal to $F\left(e\left(w_{t+1}^e\right)\right)$. Notice that $e\left(w_{t+1}^e\right)$ as well as $F\left(e\left(w_{t+1}^e\right)\right)$ are upward sloping in $w_{t+1}^e$ as $U\left(1-e, w\right)$ is downward sloping in $e$ and upward sloping in $w$.

Clearly, the decision whether to invest in schooling depends upon the expected wage. In this paper, we assume that there are several ways in which agents may predict the wage rate. Let there be $K$ different of these predictors $w_{k,t+1}$, $k = 1, 2, \ldots, K$. Since information requirements of the different predictors will differ, it seems reasonable to assume that different amounts of information costs have to be paid for these predictors. In particular, the more sophisticated predictors will require more time and effort to implement than very simple forecasts, for example because agents have to consult job counsellors or collect and process information on labor market forecasts. Each member of the population uses one of the predictors. The size of the fraction of the population of consumers using a predictor depends, apart from the predictor's information costs, upon the past success of this predictor. Hence, *intergenerational spill-overs* drive the belief formation process. In fact, the predictor choice of the new generation will depend on the level of (dis)satisfaction with the forecast methods of agents from the previous generation. Let $n_{kt}$ denote the fraction of the population using predictor $k$ in period $t$. Clearly, we must have $\sum_{k=1}^{K} n_{kt} = 1$. Assuming that beliefs are independently distributed across the population of consumers, total enrollment in schooling in period $t$ will be given by

$$E_t = E(w_{1,t+1}, w_{2,t+1}, \ldots, w_{K,t+1}) = \sum_{k=1}^{K} n_{kt} F\left(e(w_{k,t+1})\right).$$

We will abstract from dropouts and will assume that everybody that enrolls in schooling in period $t$, indeed acquires the necessary skills and supplies his labor to the market for high-skilled labor in period $t+1$. Total supply of high-skilled labor in period $t+1$ will therefore be

$$l^s\left(w_{1,t+1}, \ldots, w_{K,t+1}\right) = \sum_{k=1}^{K} n_{kt} F\left(e(w_{k,t+1})\right).$$

## 2.3 Market equilibrium

The next step is to investigate equilibrium on the market for high-skilled labor. The market clearing condition for period $t+1$ becomes

$$l^d\left(w_{t+1}\right) = \sum_{k=1}^{K} n_{kt} F\left(e\left(w_{k,t+1}\right)\right). \tag{1}$$

Denote by $w^*$ the unique solution to $l^d(w) = F(e(w))$. Furthermore, let $l_w^d \equiv \frac{\partial l^d}{\partial w}(w^*) < 0$ and $l_w^s \equiv F'(e)\, e'(w^*) > 0$ correspond to the slopes of the labor demand curve and the labor supply curve, respectively, evaluated at the steady state. The relative sizes of these derivatives play an important role in the dynamics of the full model, as will become clear shortly. Equation (1) implicitly defines the market equilibrium wage in period $t+1$ as a function of the set of predictors $w_{k,t+1}$ and corresponding fractions $n_{kt}$ as $w_{t+1} = G\left(w_{1,t+1}, \ldots, w_{K,t+1}, n_{1t}, \ldots, n_{Kt}\right)$.[2]

Let us now try to determine whether a solution $w_{t+1}$ to the market clearing condition (1) always exists. Notice that a wedge will be driven between enrollments in education in period $t$ and supply of high-skilled labor in period $t+1$ if the realized wage in period $t+1$ is not larger than the wage for low-skilled labor, that is, when $w_{t+1} \leq 1$. If $w_{t+1} < 1$, even the high-skilled workers will supply

[2] If one of the predictors corresponds to rational expectations or perfect foresight (as in the next section), i.e. $w_{t+1}^e = w_{t+1}$, the realized market equilibrium wage also turns up in the right hand side of the market equilibrium condition (1).

their labor to sector $L$ (where the demand for labor is perfectly elastic). Supply of high-skilled labor will then be 0. If $w_{t+1} > 1$ total supply of high-skilled labor will be $E_t \geq 0$ (which is independent of $w_{t+1}$ if none of the predictors corresponds to rational expectations and upward sloping if some of the agents are rational). Finally, if $w_{t+1} = 1$ high-skilled workers are indifferent between supplying their labor to sector $H$ or sector $L$. It follows that, given that $l^d(w_{t+1})$ is downward sloping, sufficient conditions for a market equilibrium to always exist are $f'(0) > 1$, and $\lim_{l \to \infty} f'(l) \leq 1$.

## 2.4 Information costs and predictor choice

A newborn generation faces the problem of choosing between different predictors. We now consider how agents decide which forecasting rule they should use. Those agents from the previous period that have not predicted the wage rate correctly might have made the wrong schooling decision. Therefore they might *regret* their decision. Together with the associated information costs this regret determines the total perceived costs of using a predictor. Notice that the probability of making the wrong decision will be lower for more accurate predictors. We assume that agents utilize the experience of the previous generation for the predictor choice. The information flow from the old generation to the newborn agents can be interpreted as learning or, as we are going to call it, *inter-generational spill-overs*. Information relevant for the predictor choice is carried over from generation to generation. One may think of the media spreading this information.

Members of the new generation compare costs of using different predictors. On the one hand, these are the costs that accrue from collecting information about future wage rates. Predicting in a more accurate way than just taking the current situation on the labor market as a cheap predictor, may require consulting employment offices or job counsellors, or finding the relevant forecasts of research institutes that have expertise. On the other hand, agents expect to face costs from using a cheaper predictor that possibly does not give the correct information on returns to education. These expected costs of an incorrect schooling decision are approximated by the *aggregate regret* of the previous generation.

For a formal treatment of the inter-generational spill-overs we have to specify the level of regret for members of the previous generation that have used predictor $k$. All agents $i$ with an effort level for schooling $e_i$ of $e_i \leq e(w_{k,t+1})$, go into schooling. After the realization of $w_{t+1}$ their ex post preferred decision is to enter schooling when $e_i \leq e(w_{t+1})$. Therefore, as long as expectations do not equal realized wages ($w_{t+1} \neq w_{k,t+1}$), some agents using predictor $k$ would have preferred to make another schooling decision. We can distinguish between two cases: *i)* $w_{t+1} < w_{k,t+1}$ and *ii)* $w_{t+1} > w_{k,t+1}$.

Figure 1 illustrates these cases for the uniform distribution. Consider the first case. Here, all agents with $e(w_{t+1}) < e_i < e(w_{k,t+1})$ will regret their decision to invest in human capital. Had they known the actual wage, they would not have spent effort on schooling. We can measure the size of their regret or dissatisfaction by comparing their actual utility with the utility they would have got, had they made the correct decision. Their actual (realized)

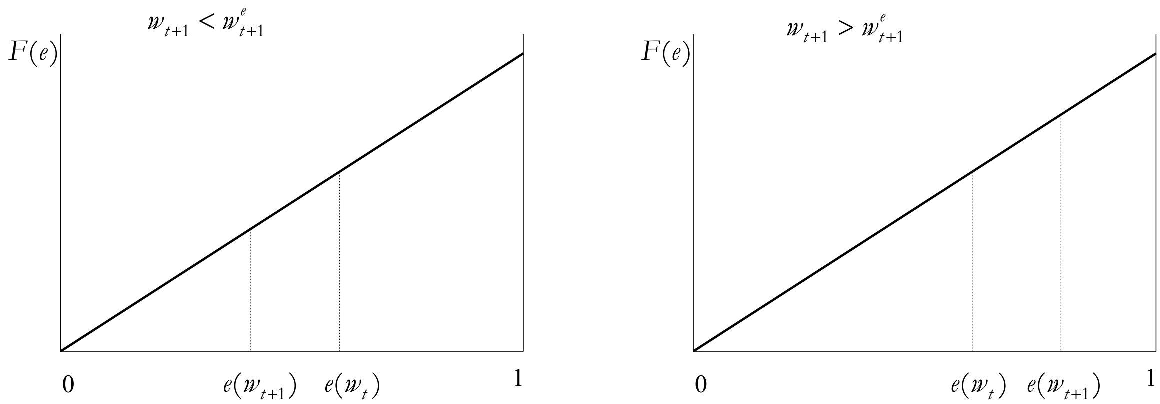


Figure 1: *Illustration of the construction of aggregate regret. Left panel: critical values of* $e\left(w_t\right)$ *and* $e\left(w_{t+1}\right)$ *for* $w_{t+1} < w_t$. *Right panel: critical values of* $e\left(w_t\right)$ *and* $e\left(w_{t+1}\right)$ *for* $w_{t+1} > w_t$.

utility will be

$$U\left(1 - e_i, w_{t+1}\right).$$

Given that the market wage is lower than they expected, they would have rather chosen not to invest into their human capital and work in the low-skill sector in the second period of their life. That choice would have given them a utility of

$$U\left(1, 1\right),$$

which we will refer to as their *potential* utility. The cost of using predictor $k$ for an agent with effort cost $e_i \in \left(e\left(w_{t+1}\right), e\left(w_{k,t+1}\right)\right)$ will therefore be the difference between potential and actual utility. That is

$$R\left(e_i, w_{t+1}\right) = U(1, 1) - U(1 - e_i, w_{t+1}).$$

It follows from $e\left(w_{t+1}\right) < e < e\left(w_{k,t+1}\right)$ that $R\left(e_i, w_{t+1}\right) > 0$. Summing up regret for all agents with predictor $k$ we find aggregate regret $R\left(w_{k,t+1}, w_{t+1}\right)$, which is

$$R\left(w_{k,t+1}, w_{t+1}\right) = \int_{e\left(w_{t+1}\right)}^{e\left(w_{k,t+1}\right)} R\left(e, w_{t+1}\right) dF\left(e\right). \tag{2}$$

Notice that $R\left(w_{t+1}, w_{t+1}\right) = 0$.

Now consider the other possibility, $w_{t+1} > w_{k,t+1}$. Then, all agents using predictor $k$ and with effort costs $e_i \in \left(e\left(w_{k,t+1}\right), e\left(w_{t+1}\right)\right)$, have decided to work in the low-skill sector in the second period of their life, but will regret this decision. They would have invested in human capital had they known the actual wage, which is now higher than they expected. We again consider the difference between potential utility, corresponding to investing in schooling, and actual utility, corresponding to working in sector $L$, which will be

$$U(1 - e_i, w_{t+1}) - U\left(1, 1\right) = -R\left(e_i, w_{t+1}\right),$$

with $e\left(w_{k,t+1}\right) < e_i < e\left(w_{t+1}\right)$. Summing up regret for all naive agents and swapping the bounds of the integral, we find aggregate regret as

$$R\left(w_{k,t+1}, w_{t+1}\right) = \int_{e\left(w_{t+1}\right)}^{e\left(w_{k,t+1}\right)} R\left(e_i, w_{t+1}\right) dF\left(e\right).$$

This expression equals the one of the first case.

Having computed aggregate regret, the final step is to determine the fraction of the newborn population that decides to use the rational predictor. We assume that agents of the newborn generation observe the set of aggregate regrets $R(w_{k,t+1}, w_{t+1})$ of the old generation. This information may be available from newspapers that report on how satisfied agents of the former generation are with their schooling decision. It may also be transferred to the new generation by socially relevant agents of the older generation. In one way or the other, newborn agents develop a feeling for the extent of regret in the society. They compare aggregate regret, which is their (naive) estimator of their own regret should they choose predictor $k$, with information costs $C_k$ associated with using predictor $k$. We suggest the following relationship for the fraction of the population using a certain rule:

$$n_{k,t+1} = H_\beta \left( R(w_{1,t+1}, w_{t+1}) + C_1, \ldots, R(w_{K,t+1}, w_{t+1}) + C_K \right). \tag{3}$$

The parameter $\beta$ tunes how fast agents respond to differences in costs.

Our full model of human capital investment with heterogeneous beliefs is now given by equations (1) and (3). In the next two sections we will investigate more specific examples of this framework. However, before going into this we will briefly discuss an alternative model where predictor choice is based upon individual regret instead of aggregate regret.

An argument for using individual regret is that private costs of effort may also influence the schooling decision. In particular, for people with a very low (high) effort level it is very likely that they will invest in schooling (not invest in schooling). Therefore their predictor choice matters less than for people with intermediate effort levels. This might lead one to use individual regret levels as the main determinant of predictor choice. This requires a slightly different set-up of the model. Now, each agent born in period $t$ belongs to a dynasty $i$. Dynasties are characterized by agents that have the same private costs of effort $e_i$. An agent born in period $t+1$ learns his effort level $e_i$, as well as the individual regret $R(e_i, w_{t+1})$ from his "parents" (compare with, for example, Orazem and Tesfatsion 1997). Now consider the case where agents can choose between the perfect foresight forecast and the naive forecast. The newborn agent uses the parents' individual regret as an estimate of his own individual regret. If the parent had perfect foresight, it informs its offspring about the individual regret he would have had with naive expectations. This individual regret is then compared with the information costs $C_k$ in order to come up with a predictor choice. Simulation with this version of the model show qualitatively similar dynamical features as the model discussed in the present paper.

# 3 Rational versus naive predictions

In the literature on demand for education two types of expectation formation processes are typically considered. Usually, human capital models assume *rational expectations* (see, for example, Becker 1975). Rational expectations imply that agents know how the economy works. Since the model is completely deterministic, rational expectations coincide with *perfect foresight*. Therefore, agents can perfectly forecast the wages, i.e. $w_{t+1}^e = w_{t+1}$. On the other hand, many contributions employ *naive expectations* (see Freeman 1986) where it is assumed that agents predict wages using the most recent wage observation, i.e. $w_{t+1}^e = w_t$. In this section, we try to merge these two approaches, by allowing agents to choose between these two prediction rules. First we will discuss the general set-up of this model and then we turn to a numerical example.

## 3.1 General set-up

Let us denote the fraction of rational forecasters in period $t$ as $n_t$. Hence, $1-n_t$ corresponds to the fraction of naive forecasters. We furthermore assume that the naive predictor can be obtained for free and that the rational predictor can be obtained at a fixed cost $C_r > 0$. The market clearing condition (1) becomes

$$l^d(w_{t+1}) = n_t F(e(w_{t+1})) + (1-n_t) F(e(w_t)). \tag{4}$$

The steady state equilibrium wage $w^*$ corresponds to the unique solution to $l^d(w) = F(e(w))$. Equation (4) implicitly defines the market clearing wage $w_{t+1}$ as a function of $w_t$ and $n_t$, say $w_{t+1} = G(w_t, n_t)$. We consider the following specification for the evolution of the fraction of rational players

$$n_{t+1} = H_\beta(C_r, R(w_t, w_{t+1})) = H(\beta(R(w_t, w_{t+1}) - C_r)),$$

where $H'(y) > 0$, $\lim_{y\to-\infty} H(y) = 0$ and $\lim_{y\to\infty} H(y) = 1$. Here $\beta$ measures how sensitive the newborn generation is with respect to the cost differential. A high value of $\beta$ implies a high level of evolutionary competition between the two forecasting rules. The full model corresponds to the following two-dimensional system of first order difference equations

$$\begin{aligned} w_{t+1} &= G(w_t, n_t), \\ n_{t+1} &= H(\beta(R(w_t, G(w_t, n_t)) - C_r)). \end{aligned} \tag{5}$$

The following proposition discusses the steady state of this dynamical system and its local stability properties.

**Proposition 1** *The steady state of the dynamical system (5) is*

$$(w^*, n^*) = (w^*, H(-\beta C_r)).$$

*This steady state is locally stable if* $\left|l_w^d\right| \geq l_w^s$. *If* $\left|l_w^d\right| < l_w^s$, *there exists a critical value* $(\beta C_r)^*$ *of* $\beta C_r$ *such that for* $\beta C_r < (\beta C_r)^*$ *the steady state is locally stable and for* $\beta C_r > (\beta C_r)^*$ *the steady state is unstable. Furthermore,* $(\beta C_r)^*$ *is implicitly given by*

$$H(-(\beta C_r)^*) = \frac{l_w^s - \left|l_w^d\right|}{2 l_w^s}.$$

*Under certain regularity conditions a* flip bifurcation *occurs for* $\beta C_r = (\beta C_r)^*$. *Then for* $\beta C_r$ *close to* $(\beta C_r)^*$ *a period two cycle exists. This period two cycle merges with the steady state at* $\beta C_r = (\beta C_r)^*$.

This result states that if the (absolute value of the) slope of the demand curve is smaller than the slope of the supply curve the steady state might become unstable. Consider what will happen in such a case. First note that in the steady state aggregate regret associated with naive expectations is zero, but there are still information costs to be paid for the rational predictor. Consumers using the naive predictor are therefore better off. Now, if the sensitivity parameter $\beta$ is not too large there will always be a sufficient amount of rational agents to stabilize the wage dynamics. However, if $\beta$ increases, which is accompanied by a rise in the level of competition between different predictors, the equilibrium value of $n$ will go down. This may destabilize the wage dynamics. At the critical value $(\beta C_r)^*$, as given in the proposition, a *period-doubling* or *flip bifurcation* occurs and a period two cycle emerges (for textbook treatments of the flip bifurcation, see Kuznetsov 1998, or Guckenheimer and Holmes 1983). This flip bifurcation may be *supercritical* or *subcritical.* For a supercritical flip bifurcation the period two cycle is stable and coexists with an unstable steady state (that is, it exists for $\beta C_r$ larger than, but close to $(\beta C_r)^*$). For a subcritical flip bifurcation the period two cycle is unstable and coexists with the stable steady state (that is, it exists for $\beta C_r$ smaller than, but close to $(\beta C_r)^*$).

For higher values of $\beta$ even more complicated behavior may occur. Let us briefly discuss the mechanism driving this complicated behavior. Suppose the system starts out close to the unstable steady state. In that case aggregate regret for using the naive predictor will be low and many people from the next generation will use the naive predictor. This destabilizes the wage dynamics and large fluctuations in wages may be observed. Aggregate regret will then go up and subsequently the fraction of rational players in the next generation will increase. This will stabilize the wage dynamics and wages will be driven to their steady state values. Simultaneously, aggregate regret for naive predictors decreases again and we end up close to the initial position. Then the whole story repeats. This mechanism shows that perpetual fluctuations can emerge naturally in a framework with an evolutionary competition between different predictors. In the next subsection we will study a numerical example where this phenomenon is indeed encountered.

## 3.2 A numerical example

### 3.2.1 The schooling decision and the labor market

We assume a standard Cobb-Douglas utility function

$$U\left(c_t, c_{t+1}\right) = c_t^{\gamma} c_{t+1}^{1-\gamma},$$

where $0 < \gamma < 1$. The marginal effort level $e^*$ is found by equating $U\left(1 - e_t, w_{t+1}^e\right)$ to $U\left(1,1\right)$. This gives

$$e_t^* = e\left(w_{t+1}^e\right) = 1 - \left(w_{t+1}^e\right)^{-\delta},$$

where $\delta \equiv (1-\gamma)/\gamma$. Furthermore we assume that individual effort costs are uniformly distributed on the unit interval: $F\left(e\right) = e$.

The production technology employed in sector $H$ is given by

$$f(l) = \frac{\alpha}{\mu} l^{\mu},$$

where $\alpha > 0$ is a productivity parameter and $0 < \mu < 1$. Note that for this production technology we indeed have $\lim_{l \to 0} f'(l) > 1$ and $\lim_{l \to \infty} f'(l) = 0 < 1$. Existence of a market clearing wage $w_{t+1}$ is therefore assured for all possible enrollment rates. Demand for high-skilled labor in period $t+1$ follows as

$$l^d(w_{t+1}) = (\frac{\alpha}{w_{t+1}})^{\frac{1}{1-\mu}}.$$

Recall that, due to the constant returns to scale technology in sector $L$, the demand for low-skilled labor is perfectly elastic.

Given that the newborn generation can choose between the rational and the naive predictor the equilibrium condition (4) becomes

$$(\frac{\alpha}{w_{t+1}})^{\frac{1}{1-\mu}} = (1-n_t)(1-w_t^{-\delta}) + n_t(1-w_{t+1}^{-\delta}). \tag{6}$$

In order to be able to explicitly solve for $w_{t+1}$ we make the following restriction on the parameters $\delta$ and $\mu$: $\delta(1-\mu) = 1$.[3] If we define $x_t \equiv w_t^{-\delta}$ then (6) is linear in $x_{t+1}$ and can be solved for $x_{t+1}$ as

$$x_{t+1} = G(x_t, n_t) = \frac{1-(1-n_t)x_t}{\alpha^\delta + n_t}.$$

The steady state equilibrium wage corresponds to the unique solution to $x^* = G(x^*, n_t)$ and is given by $x^* = \left(1+\alpha^\delta\right)^{-1} < 1$, or $w^* = \left(1+\alpha^\delta\right)^{\frac{1}{\delta}} > 1$. Furthermore, we have $l_w^s = \delta(w^*)^{-\delta-1}$ and $l_w^d = -\delta\alpha^\delta(w^*)^{-\delta-1}$, implying that the steady state will be unstable for sufficiently small $n^*$ if $\alpha < 1$.

Notice that for $x_t$ and $n_t$ satisfying $x_t < \left(1-\alpha^\delta - n_t\right)/(1-n_t)$ we have $x_{t+1} > 1$, implying $w_{t+1} < 1$. Rational agents will foresee this development and only some of the naive agents will invest in schooling in period $t$. At a wage rate for high-skilled labor lower than 1, the high-skilled agents prefer to work in sector $L$. This drives a wedge between enrollment in period $t$ and supply of high-skilled labor in period $t+1$. The supply of high-skilled labor then falls short of demand which will drive up the wage for high-skilled labor. This wage will increase exactly to the point where it equals the wage in the low-skill sector. Therefore, the actual development of $x_{t+1}$ should be written as

$$x_{t+1} = \min\left\{\frac{1-(1-n_t)x_t}{\alpha^\delta + n_t}, 1\right\}.$$

For the cases we consider in this section, the restriction on the wage for high-skilled labor never turns out to be binding.

### 3.2.2 Information costs and predictor choice

The model is closed by the specification of the predictor choice. For this we need to compute aggregate regret of naive agents (individual and aggregate

[3]Note that this requires that $\delta > 1$ or $\gamma < \frac{1}{2}$. For $\gamma > 1/2$ we cannot explicitly determine the market clearing wage $w_{t+1}$ as a function of $w_t$ and $n_t$. However, it will be shown later on that instability of the steady state and periodic behavior can also occur in this case.

regret of rational agents will always be zero). Individual regret of naive agents is given by

$$R\left(e_i, w_{t+1}\right) = U\left(1,1\right) - U\left(1-e_i, w_{t+1}\right) = 1 - \left(1-e_i\right)^{\gamma} w_{t+1}^{1-\gamma}.$$

Aggregate regret becomes

$$\begin{aligned} R\left(w_t, w_{t+1}\right) &= \int_{e(w_{t+1})}^{e(w_t)} R\left(e_i, w_{t+1}\right) dF\left(e\right) \\ &= \int_{1-w_{t+1}^{-\delta}}^{1-w_t^{-\delta}} \left[1 - \left(1-e_i\right)^{\gamma} w_{t+1}^{1-\gamma}\right] de \\ &= \left(w_{t+1}^{-\delta} - w_t^{-\delta}\right) - \frac{1}{1+\gamma} w_{t+1}^{1-\gamma} \left(w_{t+1}^{-\delta(1+\gamma)} - w_t^{-\delta(1+\gamma)}\right). \end{aligned}$$

This can again be written in terms of $x_t = w_t^{-\delta}$, giving

$$R\left(x_t, x_{t+1}\right) = \left(x_{t+1} - x_t\right) - \frac{1+\delta}{2+\delta} x_{t+1}^{-\frac{1}{1+\delta}} \left(x_{t+1}^{\frac{2+\delta}{1+\delta}} - x_t^{\frac{2+\delta}{1+\delta}}\right).$$

Now that we have determined aggregate regret, all that remains is to specify the function $H\left(.\right)$. In this paper we will make use of the so-called discrete choice model (see Brock and Hommes 1997), which gives

$$n_{t+1} = \frac{1}{1 + \exp\left[-\beta\left(R\left(x_{t+1}, x_t\right) - C_r\right)\right]}.$$

Other choices for $H\left(.\right)$ give the same qualitative behavior of the dynamics.

### 3.2.3 The full model: theoretical and numerical analysis

The full model is given by

$$\begin{aligned} x_{t+1} &= G\left(x_t, n_t\right) = \frac{1 - \left(1-n_t\right) x_t}{\alpha^{\delta} + n_t}, && (7) \\ n_{t+1} &= H\left(x_t, n_t\right) = \frac{1}{1 + \exp\left[-\beta\left(R\left(G\left(x_t, n_t\right), x_t\right) - C_r\right)\right]}. \end{aligned}$$

The steady state and the local stability properties of the steady state are discussed in the following proposition, which is a straightforward application of Proposition 1.[4]

**Proposition 2** *Consider dynamical system (7). The steady state is given by* $\left(x^*, n^*\right) = \left(\frac{1}{1+\alpha^{\delta}}, \frac{1}{1+\exp(\beta C_r)}\right)$. *For* $\alpha > 1$, *this steady state is always locally stable. For* $\alpha < 1$, *the steady state is locally stable (unstable) for* $\beta C_r$ *smaller (larger) than* $\left(\beta C_r\right)^* = \ln\left(\frac{1+\alpha^{\delta}}{1-\alpha^{\delta}}\right)$. *At* $\beta C_r = \left(\beta C_r\right)^*$ *a flip bifurcation occurs.*

[4]For the case $\gamma > 1/2$ it is not possible to explicitly solve for the equilibrium wage, but we might also have instability and periodic solutions, for $\beta C_r$ sufficiently high. Consider $\gamma = \mu = \frac{1}{2}$. It can easily be checked that the steady state equilibrium is then given by $w^* = \frac{1}{2} + \frac{1}{2}\sqrt{1+4\alpha^2}$ and is unstable under naive expectations if and only if $\alpha < \frac{1}{2}\sqrt{3}$. By a continuity argument we have that for $\alpha < \frac{1}{2}\sqrt{3}$ there exist an open neighborhood of $\left(\frac{1}{2}, \frac{1}{2}\right)$, such that for all $\left(\mu, \gamma\right)$ in this neighborhood the steady state is unstable under naive expectations. Therefore, the steady state will also be unstable in the full model when the level of evolutionary competition becomes sufficiently high.

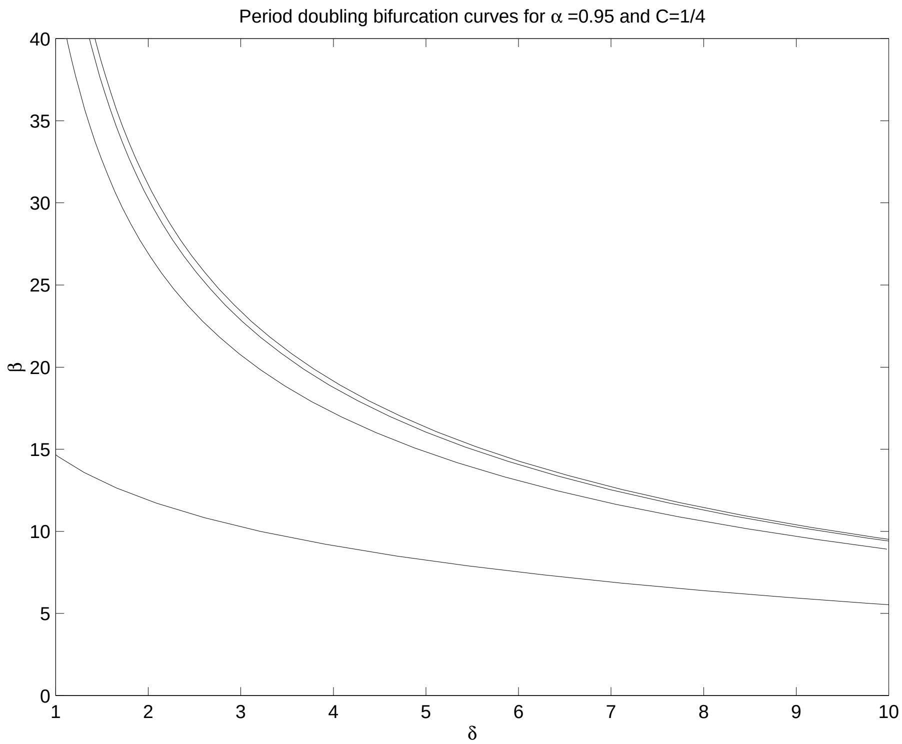


Figure 2: *Period doubling bifurcation curves for the case of rational versus naive forecasters with* $\alpha = 0.95$ *and* $C_r = \frac{1}{4}$.

Simulations suggest that the flip bifurcation occurring in this system is supercritical and a stable period two cycle exists for $\beta C_r$ larger than, but close to $(\beta C_r)^*$. If $\beta C_r$ increases further, this period two cycle might lose stability through a supercritical flip bifurcation of the second iterate of (7), leading to a stable period four cycle. This period four cycle also loses stability through a flip bifurcation and a whole cascade of period doubling bifurcations follows.

Figure 2 presents some period doubling *bifurcation curves*, i.e. curves in the parameter space along which a bifurcation of the dynamical system occurs.[5] In Figure 2 the parameters $\beta$ and $\delta$ are varied, whereas the other parameters are kept fixed at $\alpha = 0.95$ and $C_r = \frac{1}{4}$. Four different curves are drawn. The lowest curve corresponds to the first period-doubling bifurcation. This bifurcation curve was already discussed in Proposition 2 and an explicit expression for it is given by $\beta^* = 4 \ln \left( \frac{1+0.95^\delta}{1-0.95^\delta} \right)$. For parameter values of $\beta$ and $\delta$ below this curve the steady state is locally stable. For parameter values of $\beta$ and $\delta$ above this curve but below the next one, the steady state is unstable and there exists a stable period two cycle. At the second bifurcation curve this period two cycle becomes unstable and a stable period four orbit is created, which becomes unstable at the third bifurcation curve, and so on. Infinitely many of these period doubling bifurcation curves can be drawn and, as should already be apparent from Figure 2, they are closer to each other the higher the periodicity

[5]These curves are computed with the LOCBIF program (Khibnik et al. 1992).

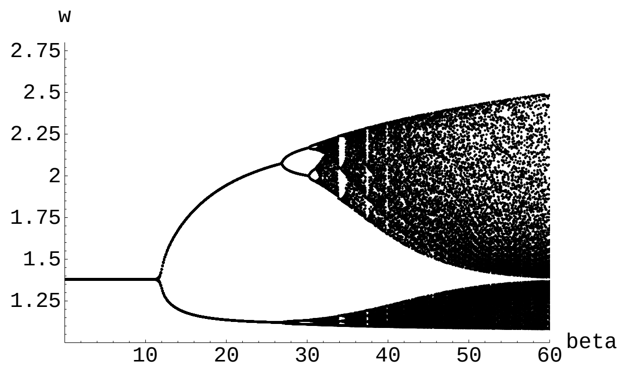


Figure 3: *Bifurcation diagram for wages, with* $\beta$ *varied from* 0 *to* 60*, for* $\alpha = 0.95$*,* $\delta = 2$ *and* $C_r = \frac{1}{4}$*.*

of the cycles.

Figure 3 shows a typical bifurcation scenario for $\alpha = 0.95$, $C_r = \frac{1}{4}$, $\delta = 2$ and different values of $\beta$. On the horizontal axis $\beta$ is varied from 0 to 60 and for each value of $\beta$ the resulting long run dynamics of wages is plotted. From this bifurcation diagram we indeed see a cascade of period doubling bifurcations eventually leading to the case where the wages move over a whole interval.

From Proposition 2 (and Figures 2 and 3) we know that the steady state loses stability at $\beta = \beta^* = 4 \ln \frac{761}{39} \approx 11.88$. The upper left panel of Figure 4 shows the period two cycle created at this first bifurcation in $(w_{t-1}, w_t)$ −space, for $\beta = 20$.[6] From Figures 2 and 3 we see that at $\beta \approx 26.91$ a period four cycle emerges, which loses stability again at $\beta \approx 30.26$. The upper right panel of Figure 4 shows this period four cycle for $\beta = 30$. These stable period two and four cycles are sometimes called *attractors*, since they attract a large set of initial states. They therefore capture the long run dynamics of the model. Besides locally stable steady states and stable periodic cycles, more complicated attractors are possible. Some examples are shown in the lower panels of Figure 4. The two-piece attractor in the lower left panel, corresponding to $\beta = 40$, emerges after a cascade of period doubling bifurcations. As $\beta$ increases further this attractor becomes larger, as can be seen in the lower right panel which shows the attractor for $\beta = 60$. Although the geometrical structure of these attractors is not very complicated, the corresponding dynamics in the wages for high-skilled labor is. Wages jump all over these attractors in an erratic way. In particular, the time series for $\beta = 40$ and $\beta = 60$ exhibit *sensitive dependence on initial conditions*. This means that two orbits generated by (7) with arbitrarily close initial conditions, will eventually follow completely different time paths. This property of the time series implies that long run predictability of wage rates is impossible, even when the system is completely deterministic. Attractors with sensitive dependence on initial conditions are sometimes called *chaotic*. The appearance of these chaotic attractors is a robust

[6]The graphs in Figure 3 are constructed as follows. System (7) is iterated 6000 times for an initial condition close to the steady state $((x_0, n_0) = (x^* + 0.01, n^*))$. Subsequently the first 1000 points are dropped and the last 5000 points are plotted in the state space $(w_{t-1}, w_t)$.

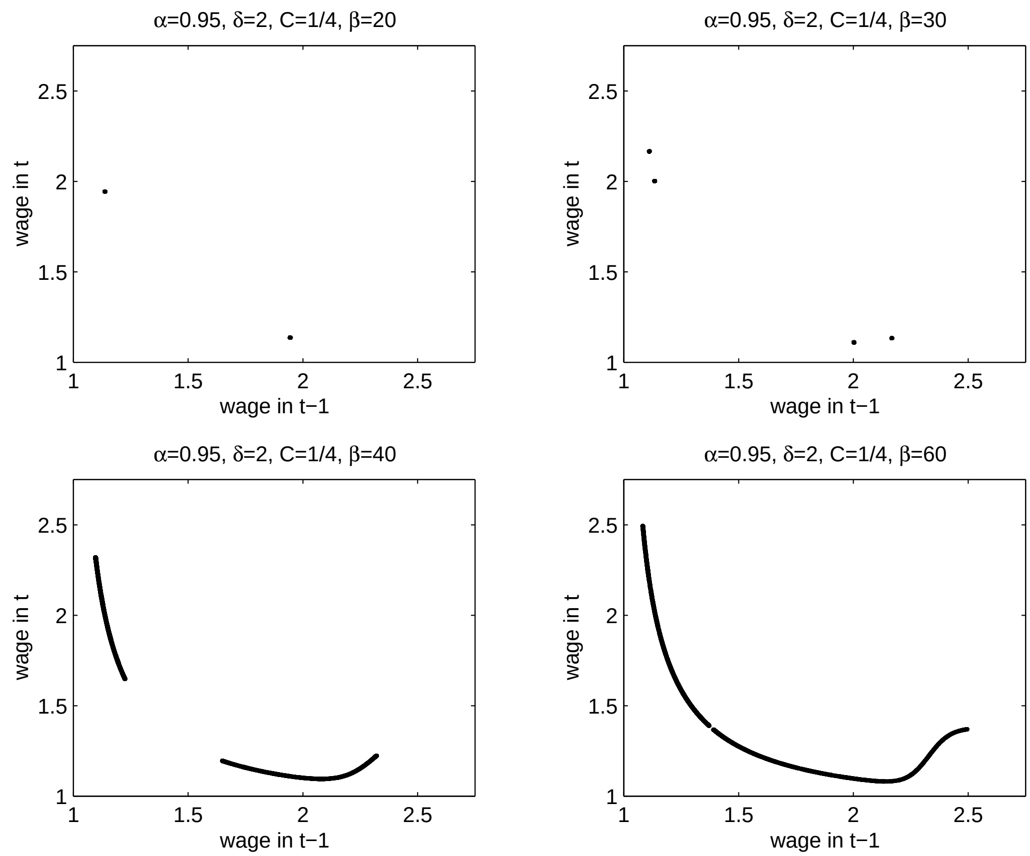


Figure 4: *Attractors in* $(w_{t-1}, w_t)$ *space for the case of rational versus naive forecasters for different values of* $\beta$ *and with* $\alpha = 0.95$, $\delta = 2$ *and* $C_r = \frac{1}{4}$. *Upper left panel:* $\beta = 20$. *Upper right panel:* $\beta = 30$. *Lower left panel:* $\beta = 40$. *Lower right panel:* $\beta = 60$.

feature of our model.

The geometrical structure of the (chaotic) attractors in Figure 4 indicates that the dynamical system (7) gives rise to a one-dimensional relationship between $w_t$ and $w_{t-1}$. The graph of this relationship is approximated by the attractor in the lower right panel of Figure 4. Note that the relationship between $w_t$ and $w_{t-1}$ is nonmonotonic. In general, a high market clearing wage in period $t-1$ leads to a low market clearing wage in period $t$ and vice versa. This finds its origin in the cobweb structure of the model. Moreover, for most values of the wage (i.e. where the attractor has a negative slope) we have that an increase in $w_{t-1}$ would lead to a decrease in $w_t$. However, for large values of $w_{t-1}$ (say $w_{t-1} > 2.2$) a small increase in $w_{t-1}$ leads to an *increase* in $w_t$. This non-monotonicity is due to the fact that if wages in period $t-1$ are very high (and wages in period $t-2$ have consequently been very low) aggregate regret will also be high which will lead a large fraction of the generation born in period $t-1$ to use the rational forecasting rule. This high value of $n_{t-1}$ will mitigate the cobweb cycle. Moreover, at the largest value of $w_{t-1}$ ($w_{t-1}$ around 2.5) the fraction of rational forecasters will be close to 1 and this brings $w_t$ close to its steady state value $w^* = \sqrt{1 + (0.95)^2} \approx 1.38$.

The shape of the attractor depends upon whether we show enrollments or wages over current wages, lagged wages, or wages one period ahead. This indicates that one would find different relationships between enrollments and relative wages when confronted with such a data set, depending on how agents' expectations are modelled empirically.

Figure 5 shows some time series for wages for high-skilled labor, the fraction of rational forecasters and the enrollment rates into higher education, for the chaotic attractor from the lower right panel in Figure 4 (i.e., $\alpha = 0.95$, $\delta = 2$, $\beta = 60$ and $C_r = \frac{1}{4}$). All variables undergo perpetual endogenous fluctuations. Although the model is deterministic, no exogenous shocks are needed to generate this persistent up and down behavior in wages and enrollments. At the beginning of these time series ($t = 100$) wages are close to their steady state value and aggregate regret will be modest. In particular, aggregate regret will be lower than the information costs for obtaining the rational forecast. Together with the high value of $\beta$, which indicates a fierce competition between the forecasting rules, this will lead to a very low fraction of rational forecasters in the next generation. However, at the steady state labor supply and demand intersect such that an 'unstable cobweb' arises and if the fraction of rational forecasters is too low, the wage dynamics will be unstable and wages will start fluctuating. These wage fluctuations lead to an increase in aggregate regret up to the point where aggregate regret is larger than the information costs for the rational forecast. When this occurs a very high fraction of the next generation will use the rational forecast, as can be seen in the sharp peak in the time series in the middle panel of Figure 5. These rational forecasters stabilize the wage dynamics and the wage for high-skilled labor will be close to the steady state again. Then the whole story repeats. Notice that the sharp peaks in the fraction of rational agents and the resulting stabilization of wages for high-skilled labor correspond to the upward sloping part of the attractor in the lower right panel of Figure 4.

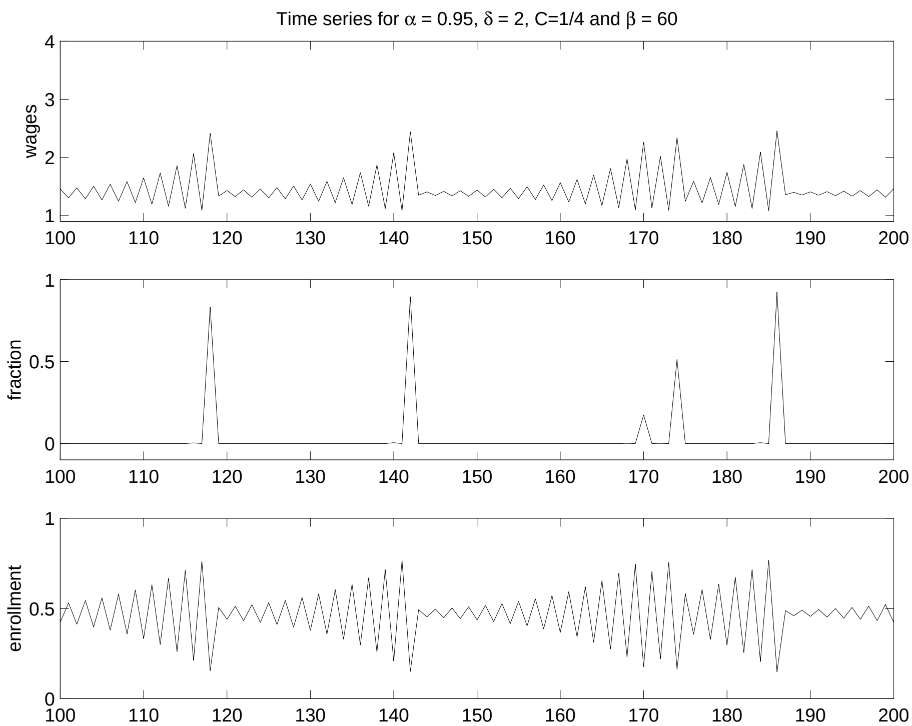


Figure 5: *Time series for wages for high-skilled labor (upper panel), fraction of rational forecasters (middle panel) and enrollment rates (lower panel) for the case of rational versus naive forecasters with* $\alpha = 0.95$*,* $\delta = 2$*,* $\beta = 60$ *and* $C_r = \frac{1}{4}$.

The example studied in this section shows that endogenous fluctuations emerge naturally in our human capital model when there is a strong evolutionary competition between different forecasting rules. The next section shows that this phenomenon is robust with respect to the set of forecasting rules under consideration.

# 4 Steady state versus adaptive forecasters

In the previous section we have seen that an evolutionary competition between perfect foresight and naive expectations in our human capital investment model naturally leads to the emergence of endogenous fluctuations in wages and enrollment rates. Although these two types of expectation formation are well-known and useful benchmarks, one might argue that they are rather special. That is, perfect foresight and naive expectations seem to correspond to the two extreme cases in expectation formation, where the former corresponds to the most sophisticated and the latter to the most unsophisticated forecasting rule. In this section we will investigate our model with two other, less extreme, forecasting rules. The newborn generation can choose between the *steady state* forecast $w^*$, which requires to predict the steady state wage differential between high- and low-skilled work, and an *adaptive* forecast, which gives a weighted average of the last two observed wage rates as a prediction for next periods wage rate. The steady state forecast is more sophisticated than the adaptive forecast, and we will assume that information costs have to be paid for obtaining it. It will be shown that the dynamical features of the system with steady state and adaptive forecasters can be qualitatively similar to those found in the previous example. Moreover, the present example might generate a type of cyclic dynamics that was not encountered in the case of perfect foresight versus naive expectations.

## 4.1 General set-up

Let $n_t$ denote the fraction of steady state forecasters and $1 - n_t$ the fraction of adaptive agents. The wage forecast of adaptive agents is

$$w_{t+1}^e = \rho w_t + (1 - \rho)\, w_{t-1},$$

where $0 \leq \rho \leq 1$. The adaptive agents therefore use a weighted average of the past two wages as their forecast of the next wage. Notice that $\rho = 1$ corresponds to the naive forecasting rule that was discussed in the previous section. Steady state forecasters use the steady state wage $w^*$ as their forecast for next period's wage. We assume that information costs $C_s > 0$ have to be paid for agents of the newborn generation to find out the steady state (the long run wage differential between high-skilled and low-skilled work). However, our results also hold for $C_s = 0$.

With these two types of forecasting rules market equilibrium (1) is given by

$$l^d\,(w_{t+1}) = (1 - n_t)\, F\,(\rho w_t + (1 - \rho)\, w_{t-1}) + n_t F\,(e\,(w^*))\,, \qquad (8)$$

which implicitly defines $w_{t+1}$ as a function of $w_t$, $w_{t-1}$ and $n_t$, say $w_{t+1} = G_s\,(w_t, w_{t-1}, n_t)$.

The fraction of steady state agents depends upon their forecasting performance relative to the forecasting performance of the adaptive agents. As long as the economy is in its steady state, the former will always have made the correct schooling decision. However, contrary to agents with perfect foresight, steady state forecasters may regret their human capital investment decision if the wage for high-skilled labor is subject to fluctuations. If the actual wage is above the steady state wage, some of the steady state forecasters that did not invest in schooling would rather have invested. If the actual wage falls short of the steady state wage, some agents regret their decision to invest in schooling. Aggregate regret for steady state agents from period $t$ is therefore given by

$$R\left(w^{*}, w_{t+1}\right)=\int_{e(w_{t+1})}^{e(w^{*})} R\left(e, w_{t+1}\right) dF\left(e\right).$$

Moreover, for adaptive expectations we find

$$R\left(w_{t}, w_{t-1}, w_{t+1}\right)=\int_{e(w_{t+1})}^{e(\rho w_{t}+(1-\rho) w_{t-1})} R\left(e, w_{t+1}\right) dF\left(e\right).$$

As in the previous example we will assume that the fraction of steady state forecasters will be driven by the difference between aggregate regret of steady state forecasters and adaptive forecasters from the previous generation. This difference is given by

$$\begin{aligned} R^{d}\left(w_{t-1}, w_{t}, w_{t+1}\right) &= R\left(w_{t-1}, w_{t}, w_{t+1}\right)-R\left(w^{*}, w_{t+1}\right) \\ &= \int_{e(w^{*})}^{e(\rho w_{t}+(1-\rho) w_{t-1})} R\left(e, w_{t+1}\right) dF\left(e\right). \end{aligned}$$

The fraction of the generation born in period $t+1$ that expects the steady state in the next period is then given by

$$n_{t+1}=H\left(\beta\left(R^{d}\left(w_{t-1}, w_{t}, w_{t+1}\right)-C_{s}\right)\right). \tag{9}$$

By introducing an auxiliary variable $v_t = w_{t-1}$, the full model, consisting of (8) and (9), can be written as the following three-dimensional system of first order difference equations

$$\begin{aligned} w_{t+1} &= G_{s}\left(v_{t}, w_{t}, n_{t}\right), \\ v_{t+1} &= w_{t}, \\ n_{t+1} &= H\left(\beta\left(R^{d}\left(v_{t}, w_{t}, G_{s}\left(v_{t}, w_{t}, n_{t}\right)\right)-C_{s}\right)\right). \end{aligned} \tag{10}$$

We will now discuss the steady state of (10) and its stability properties. First notice that if all agents are steady state forecasters the wage will be equal to the steady state wage. On the other hand, if everybody uses adaptive wage forecasts then the steady state might be unstable if the slope of the demand curve is smaller, in absolute value, than the slope of the supply curve, i.e. if $\left|l_w^d\right| < l_w^s$. However, this is not a sufficient condition for instability since, for intermediate values of $\rho$, the fluctuations in predicted wages (and therefore in enrollment rates) will be smaller than fluctuations in actual wages. Therefore, for intermediate values of $\rho$, adaptive expectations seem to have 'better' stability properties than naive expectations.

Now consider the full model. The steady state of (10) and its stability properties are summarized in three propositions. The first proposition gives the steady state and shows that this steady state is stable when $\left|l_w^d\right| \geq l_w^s$.

**Proposition 3** *The steady state of dynamical system (10) is*

$$(w^*, w^*, n^*) = (w^*, w^*, H(-\beta C_s)).$$

*If $\left|l_w^d\right| \geq l_w^s$, the steady state is locally stable.*

This result is rather intuitive. If the steady state is stable under naive expectations, one should also expect it to be stable when there is an evolutionary competition between forecasting rules that have better stability properties than naive expectations. The next step is to look at the more interesting case where the demand and supply curves are such that the steady state would be unstable under naive expectations, that is, $\left|l_w^d\right| < l_w^s$. The value of $\rho$ plays an important role here and we consider two separate cases: $\rho > \frac{2}{3}$ and $\rho < \frac{2}{3}$. The next proposition deals with the first case.

**Proposition 4** *Let $\left|l_w^d\right| < l_w^s$ and $\rho > \frac{2}{3}$. Let $n^F$ be given by*

$$n^F(\rho) = \frac{l_w^d + (2\rho - 1) l_w^s}{(2\rho - 1) l_w^s}.$$

*With respect to the stability of the steady state of (10) we have*

1. *if $\frac{2}{3} < \rho \leq \frac{1}{2}\left(1 + \left|l_w^d\right| / l_w^s\right)$ the steady state is locally stable for all values of $\beta C_s$,*
2. *if $\rho > \frac{1}{2}\left(1 + \left|l_w^d\right| / l_w^s\right)$, there exists a value $(\beta C_s)^*$ of $(\beta C_s)$, corresponding to $n^* = n^F(\rho)$, such that the steady state is locally stable for $\beta C_s < (\beta C_s)^F$, and unstable for $\beta C_s > (\beta C_s)^F$. For $\beta C_s$ in the neighborhood of $(\beta C_s)^F$ a period two cycle exists, which merges with the steady state at $\beta C_s = (\beta C_s)^F$.*

This proposition states that, for $\rho > \frac{2}{3}$ and for a sufficiently high value of $\beta$ and/or $C_s$ a flip bifurcation occurs, leading to a period two cycle. This is similar to what we have seen in the previous section, where we considered an evolutionary competition between perfect foresight and naive expectations. The case of steady state forecasters versus naive forecasters is a special case ($\rho = 1$) of the current model. The stability analysis of that case follows as a corollary to Proposition 4.

**Corollary 5** *Consider the model with steady state forecasters versus naive forecasters and let $\left|l_w^d\right| < l_w^s$. Then there exists a critical value $(\beta C_s)^*$ of $\beta C_s$ such that for $\beta C_s < (\beta C_s)^*$ the steady state is locally stable and for $\beta C_s > (\beta C_s)^*$ the steady state is unstable. Furthermore, $(\beta C_s)^*$ is implicitly given by*

$$H(-(\beta C_s)^*) = \frac{l_w^s - \left|l_w^d\right|}{l_w^s}.$$

Notice that there is a subtle difference between the bifurcation value for $\beta C_s$, when agents can choose between the steady state forecast and the naive forecast, and the bifurcation value for $\beta C_r$, when agents can choose between perfect foresight and the naive forecast, as given in Proposition 1. Actually, the critical value of the fraction of rational forecasters at the steady state is twice as small as the critical value of the fraction of steady state forecasters. On the other hand, we may assume that the costs for obtaining the rational predictor are higher than the costs for obtaining the steady state predictor. Therefore, it seems reasonable to assume that the fraction of perfect foresight agents indeed tends to be smaller than the fraction of steady state agents in the steady state.

The following proposition deals with the case $\rho < \frac{2}{3}$.

**Proposition 6** *Let* $\left|l_w^d\right| < l_w^s$ *and* $\rho < \frac{2}{3}$. *Let* $n^{NS}$ *be given by*

$$n^{NS}(\rho) = \frac{l_w^d + (1-\rho)\, l_w^s}{(1-\rho)\, l_w^s}.$$

*With respect to the stability of the steady state of (10) we have*

1. *if* $1 + \frac{l_w^d}{l_w^s} \leq \rho < \frac{2}{3}$ *the steady state is locally stable for all values of* $\beta C_s$,
2. *if* $\rho < 1 + \frac{l_w^d}{l_w^s}$ *there exists a value* $(\beta C_s)^{NS}$ *of* $(\beta C_s)$, *corresponding to* $n^* = n^{NS}(\rho)$, *such that the steady state is locally stable for* $\beta C_s < (\beta C_s)^{NS}$, *and unstable for* $\beta C_s > (\beta C_s)^{NS}$. *For* $\beta C_s$ *in the neighborhood of* $(\beta C_s)^{NS}$ *an invariant closed curve exists, which coalesces with the steady state at* $\beta C_s = (\beta C_s)^{NS}$.

Clearly, the case with $\rho < \frac{2}{3}$ is very different from the case with $\rho > \frac{2}{3}$. The difference lies in the fact that in the former case the relevant eigenvalues of the Jacobian matrix of (10), evaluated at the steady state, are complex conjugates and cross the unit circle when $\beta C_s = (\beta C_s)^{NS}$. This corresponds to a so-called *Neimark-Sacker* or *Hopf* bifurcation (for a textbook treatment of the Neimark-Sacker bifurcation, see Kuznetsov 1998).

The final case we should mention is $\rho = \frac{2}{3}$. For this value of $\rho$ we have that, at $n^* = 1 + 3l_w^d/l_w^s$, the two relevant eigenvalues are real and both are equal to $-1$. This is a so-called *strong resonance* $1:2$ *Neimark-Sacker* bifurcation. At such a bifurcation many interesting phenomena may occur and the behavior of the dynamical system near such a bifurcation can be rather complicated (see Kuznetsov 1998).

Figure 6 shows some graphs illustrating the different dynamical scenarios that are described in the propositions. Each graph gives the curves $n^F(\rho)$ and $n^{NS}(\rho)$ for a different value of the ratio of the slopes $l_w^s/l_w^d$. The dotted line in each of the graphs gives the combinations of $n^*$ and $\rho$ for which the relevant eigenvalues become complex (combinations of $n^*$ and $\rho$ to the left and above this curve give rise to complex eigenvalues). The curves denoted $n^{NS}$ and $n^F$ show the value of $n^*$ at which a Neimark-Sacker or a flip bifurcation occurs, respectively. These figures illustrate what happens in the evolutionary model for different values of the steady state fraction of steady state agents $n^*$, the belief parameter $\rho$ and the relatives slopes of the labor demand and labor

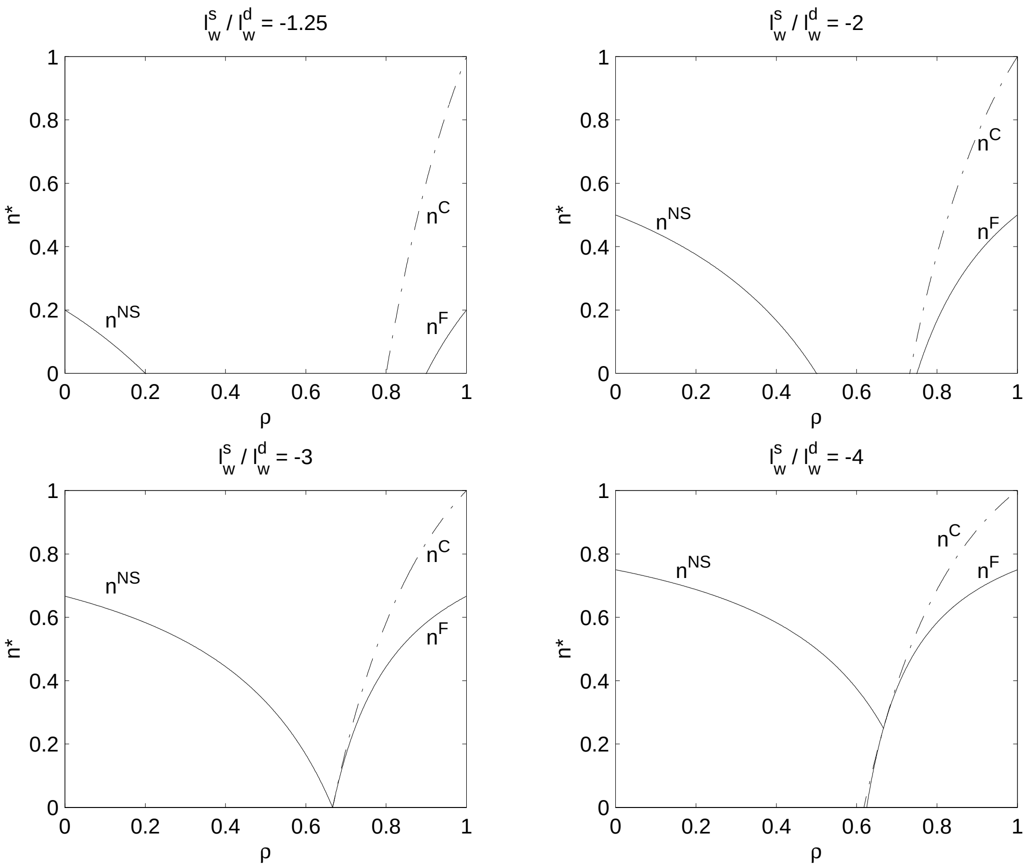


Figure 6: *Stability regions and bifurcation curves for the case of steady state versus adaptive forecasters. The horizontal axis gives the value of the parameter $\rho$ from the adaptive forecast, the vertical axis gives the fraction of steady state forecasters $n^*$ in the steady state. The dotted line $\left(n^C\right)$ gives combinations of $\rho$ and $n^*$ for which the relevant eigenvalues of the Jacobian matrix become complex, the solid lines denoted by $n^F$ and $n^{NS}$ give combinations of $\rho$ and $n^*$ for which a flip bifurcation and a Neimark-Sacker bifurcation occur, respectively. Upper left panel: $\frac{l_w^s}{l_w^d} = -1.25$. Upper right panel: $\frac{l_w^s}{l_w^d} = -2$. Lower left panel: $\frac{l_w^s}{l_w^d} = -3$. Lower right panel: $\frac{l_w^s}{l_w^d} = -4$.*

supply curve $l_w^d/l_w^s$. Notice that the steady state tends to be "more stable" for intermediate values of $\rho$, in the sense that it requires a higher degree of evolutionary competition, as measured by $\beta C_s$ or $n^*$, to destabilize the wage dynamics for these intermediate values. Also observe that $n^F(1) = n^{NS}(0) = 1 + \frac{l_w^d}{l_w^s}$ and hence the stability properties for the extreme cases $\rho = 1$ (naive expectations) and $\rho = 0$ are the same. Note, that in these two special cases the system looses its stability in different ways. Finally note that for the lower panels in Figure 6 the critical values $n^F$ and $n^{NS}$ are higher than $\frac{1}{2}$, for $\rho$ close to 0 or 1. This implies that, if we make the natural assumption that $H(0) = \frac{1}{2}$, the steady state will be unstable for these values of $\rho$, even if there are no information costs to obtain the steady state predictor, $C_s = 0$.

## 4.2 Numerical example

As before we consider a Cobb-Douglas utility function and a uniform distribution of effort costs. The market equilibrium condition (8) becomes

$$\left(\frac{\alpha}{w_{t+1}}\right)^{\frac{1}{1-\mu}} = (1-n_t)\left(1 - (\rho w_t + (1-\rho)w_{t-1})^{-\delta}\right) + n_t\left(1 - (w^*)^{-\delta}\right).$$

Notice that we can explicitly determine $w_{t+1}$ as a function of $w_t$ and $n_t$ for all admissible values of $\delta$ and $\mu$. However, in order to be able to compare this case with the case of perfect foresighted versus naive agents we will maintain the assumption $\delta(1-\mu) = 1$. We then define $x_t = w_t^{-\delta}$ again and find

$$\begin{aligned} x_{t+1} &= G_s(x_t, x_{t-1}, n_t) \\ &= \frac{1}{\alpha^\delta}(1-n_t)\left(1 - (\rho x_t^{-\frac{1}{\delta}} + (1-\rho)x_{t-1}^{-\frac{1}{\delta}})^{-\delta}\right) + \frac{1}{1+\alpha^\delta}n_t. \end{aligned} \tag{11}$$

Regret in terms of $x_t$ is easily found to be

$$\begin{aligned} R^d(x_{t-1}, x_t, x_{t+1}) = &\left(x^* - (\rho x_t^{-\frac{1}{\delta}} + (1-\rho)x_{t-1}^{-\frac{1}{\delta}})^{-\delta}\right) \\ &- \frac{1+\delta}{2+\delta}x_{t+1}^{-\frac{1}{1+\delta}}\left((x^*)^{\frac{2+\delta}{1+\delta}} - (\rho x_t^{-\frac{1}{\delta}} + (1-\rho)x_{t-1}^{-\frac{1}{\delta}})^{-\delta\frac{2+\delta}{1+\delta}}\right). \end{aligned}$$

The full model, written as a system of first order difference equations, becomes

$$\begin{aligned} x_{t+1} &= \frac{1}{\alpha^\delta}(1-n_t)\left(1 - (\rho x_t^{-\frac{1}{\delta}} + (1-\rho)y_t^{-\frac{1}{\delta}})^{-\delta}\right) + \frac{1}{1+\alpha^\delta}n_t \\ y_{t+1} &= x_t \\ n_{t+1} &= \frac{1}{1+\exp\left[-\beta\left(R^d(x_t, y_t, G_s(x_t, y_t, n_t)) - C_s\right)\right]}. \end{aligned} \tag{12}$$

The following result is a straightforward application of Proposition 3.

**Proposition 7** *Consider dynamical system (12). The steady state is given by* $(x^*, n^*) = \left(\frac{1}{1+\alpha^\delta}, \frac{1}{1+\exp(\beta C)}\right)$. *For* $\alpha \geq 1$ *the steady state is locally stable for all values of* $\beta C_s$. *For* $\alpha < 1$, *we have the following:*

1. *if* $\rho \in \left(\frac{2}{3}, \frac{1}{2}\left(1+\alpha^{\delta}\right)\right] \cup \left[1-\alpha^{\delta}, \frac{2}{3}\right)$ *the steady state is locally stable for all values of* $\beta C_s$*;*
2. *if* $\rho > \frac{1}{2}\left(1+\alpha^{\delta}\right) > \frac{2}{3}$ *the steady is locally stable (unstable) for* $\beta C_s < (>) \left(\beta C_s\right)^F$*, where*

$$\left(\beta C_s\right)^F = \ln\left(\frac{\alpha^{\delta}}{2\rho - (1+\alpha^{\delta})}\right).$$

*Under certain regularity conditions a flip bifurcation occurs at* $\beta C_s = \left(\beta C_s\right)^F$*.*

3. *if* $\rho < 1-\alpha^{\delta}$ *the steady is locally stable (unstable) for* $\beta C_s < (>) \left(\beta C_s\right)^{NS}$*, where*

$$\left(\beta C_s\right)^{NS} = \ln\left(\frac{\alpha^{\delta}}{1-\rho-\alpha^{\delta}}\right).$$

*Under certain regularity conditions a Neimark-Sacker bifurcation occurs at* $\beta C_s = \left(\beta C_s\right)^{NS}$*.*

We will now discuss the different dynamical features of system (12). Let us start out with the model with steady state forecasters versus naive forecasters ($\rho = 1$). We can compare this to the case, studied in Section 3, of perfect foresight versus naive forecasters. The upper left panel of Figure 7 shows the attractor for the case of steady state versus naive forecasters, with $\alpha = 0.95$, $\delta = 2$, $C = \frac{1}{4}$ and $\beta = 60$. Note the resemblance with the attractor in the lower right panel in Figure 4. The times series of this model are also very similar to those of the model studied in Section 3. The dynamics for the two different models seem to be closely connected. This holds for other parameter choices as well. The other panels in Figure 7 show attractors for parameter values $\alpha = \frac{1}{2}$, $\delta = 2$ and $C_s = 1/4$. We then have $\left|l_w^d / l_w^s\right| = 4$, which corresponds to the lower right panel of Figure 6. (Note that the steady state will be unstable for any positive value of $\beta C_s$ when $\rho \leq 1/2$ or $\rho \geq 3/4$.) First let $\rho = 7/10$. From Proposition 4 we know that at $\beta = \beta^F = 4\ln(5/3) \approx 2.043$ a flip bifurcation occurs and a period two orbit is created. For $\beta$ slightly larger than $\beta^F$, wages for high-skilled labor will then move in a period two cycle. This period two cycle is shown for $\beta = 2.08$ in the upper right panel in Figure 7. The lower left panel in Figure 7 deals with the case $\rho = \frac{6}{10}$. From Proposition 6 we know that this gives rise to complex eigenvalues which cross the unit circle when $\beta = \beta^{NS} = 4\ln(5/3) \approx 2.043$. At this Neimark-Sacker bifurcation an invariant closed curve is created. For $\rho = 6/10$ this Neimark-Sacker bifurcation is *supercritical*, which means that for $\beta$ close to, but larger than $\beta^{NS}$, an attracting closed curve exists. This attracting closed curve is shown for $\beta = 2.08$ in the lower left panel of Figure 7. Wages for high-skilled labor then move over this closed curve. The dynamics on this curve may be periodic or quasi-periodic. Generically, as $\beta$ increases, the system undergoes numerous bifurcations resulting in periodic orbits on the closed curve. Then two periodic orbits, of the same period, arise on the closed curve and the dynamics is then periodic (this periodic orbit may however have a very long period). If for a certain value of $\beta$ such a periodic orbit does not exist the dynamics on the closed curve is quasi-periodic: the orbit of each point on the closed curve comes back arbitrarily close to that

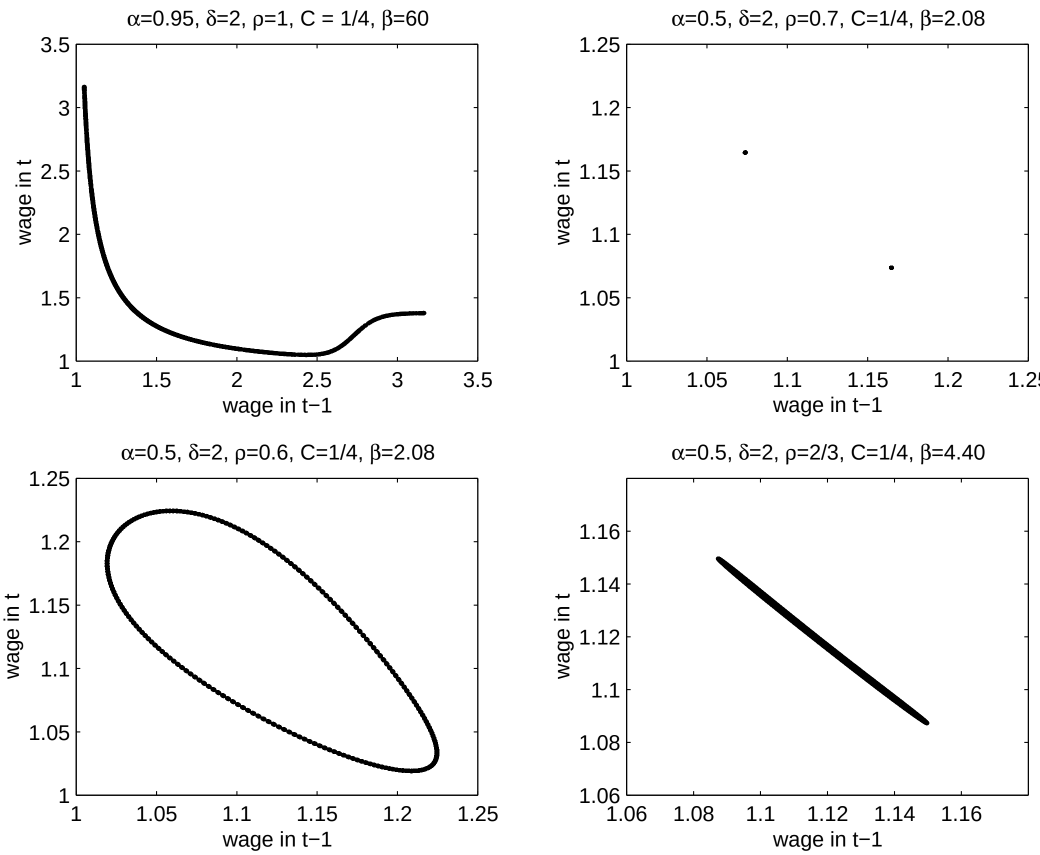


Figure 7: *Attractors in $(w_{t-1}, w_t)$ space for the case of steady state versus adaptive forecasters. Upper left panel: $\alpha = 0.95$, $\delta = 2$, $\rho = 1$, $\beta = 60$ and $C_s = \frac{1}{4}$. Upper right panel: $\alpha = 0.5$, $\delta = 2$, $\rho = 0.7$, $\beta = 2.08$ and $C_s = \frac{1}{4}$. Lower left panel: $\alpha = 0.5$, $\delta = 2$, $\rho = 0.6$, $\beta = 2.08$ and $C_s = \frac{1}{4}$. Lower right panel: $\alpha = 0.5$, $\delta = 2$, $\rho = 2/3$, $\beta = 4.4$ and $C_s = \frac{1}{4}$.*

point but not exactly. The orbit fills up the closed curve completely. The time series of such an orbit is almost but not exactly periodic.

A *subcritical Neimark-Sacker* bifurcation also occurs in this model. At such a subcritical Neimark-Sacker bifurcation the invariant closed curve exists for $\beta < \beta^{NS}$ and is repelling. Then all orbits starting out in the interior of the closed curve will converge to the steady state and all orbits starting outside the closed curve will diverge. The invariant closed curve therefore gives the border of the basin of attraction of the locally stable steady state. As $\beta$ approaches $\beta^{NS}$, the closed curve (and therefore the basin of attraction of the steady state) shrinks until it merges with the steady state at $\beta = \beta^{NS}$. For $\beta > \beta^{NS}$ a closed curve does not exist and the steady state is unstable. Due to the restriction on the wage for high-skilled labor the wage dynamics will converge to a period two orbit that cycles between a high wage and a wage of 1. These subcritical Neimark-Sacker bifurcations occur for $\rho < \rho^0 \approx 0.514$. For $\rho > \rho^0$ the Neimark-Sacker bifurcation is supercritical and leads to an attracting invariant closed curve as discussed above. At the parameter value

$(\rho, \beta) = \left(\rho^0, -4\ln\left(3 - 4\rho^0\right)\right)$ our dynamical system undergoes a *Chenciner* bifurcation. This implies that there is a curve in the parameter space, starting at the Chenciner bifurcation point and going down in $(\rho, n^*)$ −space, where the unstable invariant closed curve, created at the subcritical Neimark-Sacker bifurcation, undergoes a *saddle-node* bifurcation and two invariant curves, one lying in the interior of the other, are created. The outer curve is unstable and the inner curve is stable. For a textbook treatment of the Chenciner bifurcation see Kuznetsov (1998). Gaunersdorfer, Hommes and Wagener (2001) also discuss the Chenciner bifurcation in some depth in an evolutionary model similar to ours. Finally, the lower right panel in Figure 7 shows the attractor created in the 1 : 2 strong resonance Neimark-Sacker bifurcation at $\rho = \frac{2}{3}$ and $\beta = 4\ln 3 \approx 4.39$. The attractor created in this bifurcation is a very "flat" invariant closed curve and is shown for $\beta = 4.4$. Typically, chaotic phenomena occur close to Chenciner and 1 : 2 strong resonance Neimark-Sacker bifurcations.

# 5 Concluding remarks

Our aim was to endogenously explain variations in enrollments to higher education. For that purpose we developed a human capital model with overlapping generations. In this model agents have the choice to invest into schooling in the first period of their life in order to earn a higher wage in the second period of their life, or not to put effort into schooling and work in the low-skill sector in both periods. Contrary to other human capital models our agents are heterogenous in their expectations on the returns to education. An evolutionary competition determines the fraction of agents making 'not-so-rational' schooling choices. As access to information is costly, 'not-so-rational' agents use current and past returns on education as a predictor for future returns, unless experience of the previous generation indicates that using a more sophisticated prediction rule is advantageous. The interplay of destabilizing backward looking expectations and a stabilizing sophisticated predictor may generate endogenous fluctuations in the demand for education. No exogenous shocks are needed to arrive at perpetual changes in enrollments. This holds true, even under standard assumptions on labor demand, which is downward sloping in our examples, and agents' preferences, which are Cobb-Douglas. We illustrated our point for two different pairs of prediction rules: rational expectations versus naive expectations in schooling choices, and steady state forecasters, which are agents who know the steady state wage differential between high-skill and low-skill jobs, versus adaptive beliefs. The emergence of complicated dynamics is robust against the competitive evolutionary beliefs that we study. The routes to interesting dynamics are characterized by various sorts of bifurcations, such as supercritical flip bifurcations, as well as subcritical and supercritical Neimark-Sacker-bifurcations.

Costs of collecting information and the degree of evolutionary pressure that arises from the dissatisfaction of previous generations' schooling decisions are key for the dynamics of our model. This suggests that policies tearing down obstacles for collecting information on returns to education may stabilize flows to higher education. The reason is that students will less likely make use of cheap and possibly destabilizing backward looking prediction rules. If it is more easy to find and process information on future labor market states, more students will be guided by the 'true' wage differentials in their schooling choice.

The model that we developed is parsimonious, mainly for reasons of tractability. Extensions of the model may include drop-outs from school, and possibly a more sophisticated life-time structure. Empirically, it would be interesting to have time series evidence on agents' expectation formation, to see whether it resembles the variation over time generated by our model.

# References


Arifovic, J. (1994): Genetic algorithm learning and the cobweb model. Journal of Economic Dynamics and Control 18, 3-28.

Bala, V. and G. Sorger (1997): The evolution of human capital in an interacting agent economy. Journal of Economic Behavior and Organization 36, 85-108.

Bala, V. and G. Sorger (2001): A spatial temporal model of human capital accumulation. Journal of Economic Theory, 153-179.

Barucci, E. (1999): Heterogenous beliefs and learning in forward looking economic models. Journal of Evolutionary Economics 9, 453-464.

Becker, G.S. (1975) Human Capital. A theoretical and empirical analysis, with special reference to education, 2nd edition. Columbia University Press. London and New York.

Betts, J.R. (1996): What do students know about wages? Evidence from a survey of undergraduates. Journal of Human Resources 31, 27-56.

Borghans, L., A. de Grip and H. Heijke (1996): Labor market information and the choice of vocational specialization. Economics of Education Review 15, 1, 59-74.

Brock, W.A. and C. Hommes (1997): A rational route to randomness. Econometrica 65, 5, 1059-1095.

Brock, W.A. and C. Hommes (1998): Heterogeneous beliefs and routes to chaos in a simple asset pricing model. Journal of Economic Dynamics and Control 22, 1235-1274.

Brunello, G., C. Lucifora and R. Winter-Ebmer (2001): The wage expectations of European College students. University of Linz, mimeo.

Card, D. and T. Lemieux (2000): Dropout and enrollment trends in the postwar period: what went wrong in the 1970's? NBER Working Paper 7658.

de la Croix, D. (2001): Growth dynamics and education spending: the role of inherited tastes and abilities. European Economic Review 45, 1415-1438.

Chiarella, C. (1992): The dynamics of speculative behaviour. Annals of Operations Research 37, 101-123.

Dawid, H. and M. Kopel (1998): On economic applications of the genetic algorithm: a model of the cobweb type. Journal of Evolutionary Economics 8, 297-315.

Droste, E., C.H. Hommes and J. Tuinstra (2002): Endogenous fluctuations under evolutionary pressure in Cournot competition. Games and Economic Behavior, forthcoming.

Duchesne, I. and W. Nonneman (1998): The demand for higher education in Belgium. Economics of Education Review 17, 2, 211-218.

Dellas, H. and P. Sakellaris (1995): On the cyclicality of the demand for education: theory and evidence. University of Maryland Working Paper No. 95-12.

Franke, R. (1998): Coevolution and stable adjustments in the cobweb model. Journal of Evolutionary Economics 8, 383-406.

Freeman, R.B. (1975a): Legal cobwebs: a recursive model of the market for new lawyers. Review of Economics and Statistics, 171-179.

Freeman, R.B. (1975b): Supply and salary adjustment to the changing science manpower market: physics. American Economic Review 65, 27-39.

Freeman, R.B. (1976a): A cobweb model of the supply and starting salary of new engineers. Industrial and Labour Relations Review, 236-248.

Freeman, R.B. (1976b): The overeducated American. Academic Press. New York, San Francisco, London.

Freeman, R.B. (1986): Demand for education, in: O. Ashenfelter and R. Layard (eds.): Handbook of Labor Economics, Vol. I, 357-386.

Gaunersdorfer, A, C.H. Hommes and F.O.O. Wagener (2001): Bifurcation routes to volatility clustering. Tinbergen Institute Discussion Paper, TI 200100-15/1.

Guckenheimer, J. and P. Holmes (1993): Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields, New York: Springer-Verlag.

Khibnik, A.I., Y.A. Kuznetsov, V.V. Levitin and E.V. Nikolaev (1992): Interactive LOCal BIFurcation analyzer: LOCBIF, version 2.

Kuznetsov, Y.A. (1998): Elements of Applied Bifurcation Theory ($2^{nd}$ed.), Springer Verlag, New York.

Lux, T. (1998): The socio-economic dynamics of speculative markets. Journal of Economic Behavior and Organization 33, 2, 143-165.

Manski, C.F. (1993): Adolescent econometricians: how do youth infer the returns to schooling? In: C.T. Clotfelter and M. Rothschild (eds.), Studies of the Supply and Demand in Higher Education. The Chicago University Press. Chicago and London. 43-60.

Neugart, M. (2001): The demand for education: enrollments of engineers in Germany. Social Science Research Center Berlin (WZB). mimeo.

Orazem, P. and L. Tesfatsion (1997): Macrodynamic implications of income-transfer policies for human capital investment and school effort. Journal of Economic Growth 2, 305-329.

Quinn, R. and J. Price (1998): The demand for medical education: an augmented human capital approach. Economics of Education Review 17, 3, 337-347.

Siow, A. (1984): Occupational choice under uncertainty. Econometrica 52, 3, 631-645.

Titze, H. unter Mitarbeit von H. Herlitz, V. Mueller-Benedict und A. Nath (1986): Datenhandbuch zur deutschen Bildungsgeschichte. Band I, Teil 1: Das Hochschulstudium in Preussen und Deutschland 1820-1941. Göttingen, Vandenhoek.

Titze, H. unter Mitarbeit von H. Herlitz, V. Mueller-Benedict und A. Nath (1993): Datenhandbuch zur deutschen Bildungsgeschichte. Band I, Teil 2: Wachstum und Differenzierung der deutschen Universitäten 1830-1945. Göttingen, Vandenhoek.

Topel, R.H. (1997): Factor proportions and relative wages: the supply-side determinants of wage inequality. Journal of Economic Perspectives 11, 2, 55-74.

Zarkin, G.A. (1983): Cobweb versus rational expectations: lessons from the market for public school teachers. Economic Letters 13, 87-95.

Zarkin, G.A. (1985): Occupational choice: an application to the market for public school teachers. Quarterly Journal of Economics 100, 409-446.

# Appendix: Proofs of main results

In this appendix we provide proofs of the main results from this paper.

**Proof of Proposition 1.** The steady state follows directly from (5) and the observations that, at a steady state, rational and naive agents have correct predictions and regret of naive agents is zero at the steady state. Stability of the steady state is determined by the eigenvalues of the Jacobian matrix evaluated at the steady state. Taking the total differential of (4) and evaluating at the equilibrium $(w^*, n^*)$ gives

$$\left.\frac{\partial G}{\partial w_t}\right|_{(w^*,n^*)} = \frac{(1-n^*)\, l_w^s}{l_w^d - n^* l_w^s} \text{ and } \left.\frac{\partial G}{\partial n_t}\right|_{(w^*,n^*)} = 0.$$

Furthermore, we have

$$\left.\frac{\partial n_{t+1}}{\partial w_t}\right|_{(w^*,n^*)} = \beta H'\left(-\beta C\right) \text{ and } \left.\frac{\partial n_{t+1}}{\partial n_t}\right|_{(w^*,n^*)} = 0.$$

The Jacobian matrix therefore becomes

$$J = \begin{pmatrix} \frac{(1-n^*) l_w^s}{l_w^d - n^* l_w^s} & 0 \\ \left.\frac{\partial n_{t+1}}{\partial w_t}\right|_{(w^*,n^*)} & 0 \end{pmatrix}$$

and has eigenvalues $\lambda_1 = \frac{(1-n^*) l_w^s}{l_w^d - n^* l_w^s}$ and $\lambda_2 = 0$. The steady state is locally stable whenever the eigenvalues lie in the unit circle which, in this case, corresponds to

$$(1-2n^*)\, l_w^s < \left| l_w^d \right|$$

If $\left| l_w^d \right| \geq l_w^s$ this condition is always satisfied. However, if $\left| l_w^d \right| < l_w^s$, then for all $n^* < \frac{l_w^s - \left| l_w^d \right|}{2 l_w^s}$, the first eigenvalue is smaller than $-1$, and the steady state is unstable. ■

**Proofs of Propositions 3, 4 and 6.** The steady state follows directly from (10) and the observations that, at a steady state, $w_t = w^*$ and $R^d\left(w^*, w^*, w^*\right) = 0$. For evaluating stability of this steady state we consider the Jacobian matrix of the linearized system, evaluated at the equilibrium. This Jacobian matrix is given by

$$J = \begin{pmatrix} (1-n^*)\, \rho \frac{l_w^s}{l_w^d} & (1-n^*)(1-\rho) \frac{l_w^s}{l_w^d} & 0 \\ 1 & 0 & 0 \\ \left.\frac{\partial n_{t+1}}{\partial w_t}\right|_{(w^*,w^*,n^*)} & \left.\frac{\partial n_{t+1}}{\partial y_t}\right|_{(w^*,w^*,n^*)} & 0 \end{pmatrix},$$

where the elements on the first row are found by totally differentiating the market equilibrium condition (8). The eigenvalues are given by

$$\lambda_{1,2} = \frac{1}{2}(1-n^*)\, \rho \frac{l_w^s}{l_w^d} \pm \frac{1}{2} \sqrt{(1-n^*) \frac{l_w^s}{l_w^d} \left( (1-n^*)\, \rho^2 \frac{l_w^s}{l_w^d} + 4(1-\rho) \right)}$$

and $\lambda_3 = 0$. First consider the case with $l_w^s < \left|l_w^d\right|$. For the absolute value of the nonzero eigenvalues we then have

$$\begin{aligned}\left|\lambda_{1,2}\right| &\leq \frac{1}{2}\left(1-n^*\right)\rho\frac{l_w^s}{\left|l_w^d\right|} + \frac{1}{2}\sqrt{\left(1-n^*\right)\frac{l_w^s}{\left|l_w^d\right|}\left(\left(1-n^*\right)\rho^2\frac{l_w^s}{\left|l_w^d\right|} + 4\left(1-\rho\right)\right)} \\ &< \frac{1}{2}\rho + \frac{1}{2}\sqrt{\left(\rho^2 + 4\left(1-\rho\right)\right)} = \frac{1}{2}\rho + \frac{1}{2}\sqrt{\left(2-\rho\right)^2} = 1,\end{aligned}$$

and hence the steady state is locally stable when $l_w^s < \left|l_w^d\right|$. Now consider the case with $l_w^s > \left|l_w^d\right|$. The nonzero eigenvalues are complex conjugates when the expression under the root is negative, which is equivalent with

$$n^* > n^c\left(\rho\right) \equiv 1 + 4\frac{\left(1-\rho\right)}{\rho^2}\frac{l_w^d}{l_w^s},$$

otherwise all eigenvalues are real. It can easily be checked that $n^c$ is increasing in $\rho$, that $n^c\left(1\right) = 1$ and that $n^c\left(\rho^a\right) = 0$ for

$$\rho^a = 2\frac{l_w^d}{l_w^s}\left(1 - \sqrt{1 - \frac{l_w^s}{l_w^d}}\right) \in \left(0, 1\right).$$

Therefore, for $\rho < \rho^a$ the eigenvalues are complex for any value of $n^*$. Now we investigate what happens when eigenvalues are real and when they are complex, respectively.

1. *Real eigenvalues.* If the eigenvalues are real the smallest eigenvalue (corresponding to the negative root) is smaller than $-1$ if and only if

$$n^* < n^F\left(\rho\right) = \frac{l_w^d + \left(2\rho - 1\right)l_w^s}{\left(2\rho - 1\right)l_w^s}.$$

   Hence for $n < n^F$ and $\rho > \rho^c$ the steady state is unstable. Notice that $n^F$ is increasing in $\rho$ and that $n^F\left(\rho^b\right) = 0$, for $\rho^b = \frac{1}{2}\left(1 - \frac{l_w^d}{l_w^s}\right) > \frac{1}{2}$. Furthermore, we have $n^c\left(\rho\right) - n^F\left(\rho\right) = -\frac{9\left(\rho - \frac{2}{3}\right)^2}{\rho^2\left(2\rho-1\right)}\frac{l_w^d}{l_w^s}$, which implies that $n^f\left(\rho\right) \leq n^c\left(\rho\right)$, for all $\rho > \frac{1}{2}$.

2. *Complex eigenvalues.* If the first two eigenvalues are complex a so-called Neumark-Sacker bifurcation occurs as the determinant of the upper $2 \times 2$ submatrix of the Jacobian equals $+1$. This happens when

$$n^{NS}\left(\rho\right) = \frac{l_w^d + \left(1-\rho\right)l_w^s}{\left(1-\rho\right)l_w^s}.$$

   Notice that $n^{NS}$ is decreasing in $\rho$ and $n^{NS}\left(\rho^c\right) = 0$ for $\rho^c = 1 + \frac{l_w^d}{l_w^s}$.

As can be easily checked, the curves $n^C, n^F$ and $n^{NS}$ intersect at $(\rho, n) = \left(\frac{2}{3}, 1 + 3\frac{l_w^d}{l_w^s}\right)$. Therefore, we find that for $\rho > \frac{2}{3}$, the system undergoes a flip bifurcation at $n^* = n^F\left(\rho\right)$ and the steady state is unstable for $n^* < n^F\left(\rho\right)$. For $\rho < \frac{2}{3}$, the system undergoes a Neimark-Sacker bifurcation at $n^* = n^{NS}\left(\rho\right)$ and the steady state is unstable for $n^* < n^{NS}\left(\rho\right)$. ■

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**Benchmarking Deutschland: Arbeitsmarkt und Beschäftigung. Bericht der Arbeitsgruppe Benchmarking und der Bertelsmann Stiftung**
2001, Berlin/Heidelberg/New York, Springer, 440 Seiten

European Academy of the Urban Environment
**New institutional arrangements in the labour market. Transitional labour markets as a new full employment concept**
1998, Berlin, EA.UE series „The Urban Environment in Europe“, 135 Seiten

Gernot Grabher, David Stark (Eds.)
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2001, Bonn, Friedrich-Ebert-Stiftung, Schriftenreihe der Abteilung „Arbeit und Sozialpolitik“, 83 Seiten

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